Exercise 1D

1 a $f(x) = 4x^3 - 5x^2 + 7x + 1$

$$f(2) = 4(2)^3 - 5(2)^2 + 7(2) + 1$$
$$= 4(8) - 5(4) + 7(2) + 1$$
$$= 32 - 20 + 14 + 1$$
$$= 27$$

b $f(x) = 2x^5 - 32x^3 + x - 10$

$$f(4) = 2(4)^5 - 32(4)^3 + 4 - 10$$
$$= 2(1024) - 32(64) - 6$$
$$= 2048 - 2048 - 6$$
$$= -6$$

c $f(x) = -2x^3 + 6x^2 + 5x - 3$

$$f(-1) = -2(-1)^3 + 6(-1)^2 + 5(-1) - 3$$
$$= -2(-1) + 6(1) + 5(-1) - 3$$
$$= 2 + 6 - 5 - 3$$
$$= 0$$

d $f(x) = 7x^3 + 6x^2 - 45x + 1$

$$f(-3) = 7(-3)^3 + 6(-3)^2 - 45(-3) + 1$$
$$= 7(-27) + 6(9) - 45(-3) + 1$$
$$= -189 + 54 + 135 + 1$$
$$= 1$$

e $f(x) = 4x^4 - 4x^2 + 8x - 1$

$$f\left(\frac{1}{2}\right) = 4x^4 - 4x^2 + 8x - 1$$
$$= 4\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) - 1$$
$$= 4\left(\frac{1}{16}\right) - 4\left(\frac{1}{4}\right) + 8\left(\frac{1}{2}\right) - 1$$
$$= \frac{1}{4} - 1 + 4 - 1$$
$$= \frac{9}{4}$$

1 f $f(x) = 243x^4 - 27x^3 - 3x + 7$

$$f\left(\frac{1}{3}\right) = 243\left(\frac{1}{3}\right)^4 - 27\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right) + 7$$
$$= 243\left(\frac{1}{81}\right) - 27\left(\frac{1}{27}\right) - 3\left(\frac{1}{3}\right) + 7$$
$$= 3 - 1 - 1 + 7$$
$$= 8$$

g $f(x) = 64x^3 + 32x^2 - 16x + 9$

$$f\left(-\frac{3}{4}\right) = 64\left(-\frac{3}{4}\right)^3 + 32\left(-\frac{3}{4}\right)^2 - 16\left(-\frac{3}{4}\right) + 9$$
$$= 64\left(-\frac{27}{64}\right) + 32\left(\frac{9}{16}\right) - 16\left(-\frac{3}{4}\right) + 9$$
$$= -27 + 18 + 12 + 9$$
$$= 12$$

h $f(x) = 81x^3 - 81x^2 + 9x + 6$

$$f\left(\frac{2}{3}\right) = 81\left(\frac{2}{3}\right)^3 - 81\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right) + 6$$
$$= 81\left(\frac{8}{27}\right) - 81\left(\frac{4}{9}\right) + 9\left(\frac{2}{3}\right) + 6$$
$$= 24 - 36 + 6 + 6$$
$$= 0$$

i $f(x) = 243x^6 - 780x^2 + 6$

$$f\left(-\frac{4}{3}\right) = 243\left(-\frac{4}{3}\right)^6 - 780\left(-\frac{4}{3}\right)^2 + 6$$
$$= 243\left(\frac{4096}{729}\right) - 780\left(\frac{16}{9}\right) + 6$$
$$= \frac{4096}{3} - \frac{4160}{3} + \frac{18}{3}$$
$$= -\frac{46}{3}$$

1 j $\mathrm{f}(x)=125x^4+5x^3-9x$

$$\mathrm{f}\left(-\frac{3}{5}\right)=125\left(-\frac{3}{5}\right)^4+5\left(-\frac{3}{5}\right)^3-9\left(-\frac{3}{5}\right)$$
$$=125\left(\frac{81}{625}\right)+5\left(-\frac{27}{125}\right)-9\left(-\frac{3}{5}\right)$$
$$=\frac{81}{5}-\frac{27}{25}+\frac{27}{5}$$
$$=\frac{513}{25}$$

2 $\mathrm{f}(x)=2x^3-3x^2-2x+a$

Since $\mathrm{f}(1)=-4$

$$2(1)^3-3(1)^2-2(1)+a=-4$$
$$2-3-2+a=-4$$
$$a=-1$$

3 $\mathrm{f}(x)=-3x^3+4x^2+bx+6$

Since $\mathrm{f}(-2)=10$

$$-3(-2)^3+4(-2)^2+b(-2)+6=10$$
$$-3(-8)+4(4)+b(-2)+6=10$$
$$24+16-2b+6=10$$
$$2b=36$$
$$b=18$$

4 $\mathrm{f}(x)=216x^3-32x^2+cx-8$

Since $\mathrm{f}\left(\frac{1}{2}\right)=1$

$$216\left(\frac{1}{2}\right)^3-32\left(\frac{1}{2}\right)^2+c\left(\frac{1}{2}\right)-8=1$$
$$216\left(\frac{1}{8}\right)-32\left(\frac{1}{4}\right)+c\left(\frac{1}{2}\right)-8=1$$
$$27-8+\frac{1}{2}c-8=1$$
$$\frac{1}{2}c=-10$$
$$c=-20$$

5 $\mathrm{f}(x)=x^6-36x^3+243$

$$\mathrm{f}(3)=(3)^6-36(3)^3+243$$
$$=729-972+243$$
$$=0$$

Since f(3) = 0, (x – 3) is a factor of f(x)

6 $\mathrm{f}(x)=2x^3+17x^2+31x-20$

$$\mathrm{f}\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+17\left(\frac{1}{2}\right)^2+31\left(\frac{1}{2}\right)-20$$
$$=2\left(\frac{1}{8}\right)+17\left(\frac{1}{4}\right)+31\left(\frac{1}{2}\right)-20$$
$$=\frac{1}{4}+\frac{17}{4}+\frac{31}{2}-20$$
$$=0$$

Since $\mathrm{f}\left(\frac{1}{2}\right)=0$, (2$x$ – 1) is a factor of f(x)

7 $\mathrm{f}(x)=x^2+3x+q$

Since $\mathrm{f}(2)=3$,

$$(2)^2+3(2)+q=3$$
$$10+q=3$$
$$q=-7$$

so

$$\mathrm{f}(x)=x^2+3x-7$$
$$\mathrm{f}(-2)=(-2)^2+3(-2)-7$$
$$=4-6-7$$
$$=-9$$

8 $\mathrm{g}(x)=x^3+ax^2+3x+6$

Since $\mathrm{g}(-1)=2$,

$$(-1)^3+a(-1)^2+3(-1)+6=2$$
$$-1+a-3+6=2$$
$$a=0$$

so

$$\mathrm{g}(x)=x^3+3x+6$$
$$\mathrm{g}\left(\frac{2}{3}\right)=\left(\frac{2}{3}\right)^3+3\left(\frac{2}{3}\right)+6$$
$$=\frac{8}{27}+2+6$$
$$=\frac{224}{27}$$

9 $\mathrm{f}(x) = 2x^3 - x^2 + ax + b$

$\mathrm{f}(2) = 14$

So

$2(2)^3 - (2)^2 + a(2) + b = 14$

$2(8) - (4) + a(2) + b = 14$

$16 - 4 + 2a + b = 14$

$2a + b = 2$ **(1)**

$\mathrm{f}(-3) = -86$

So

$2(-3)^3 - (-3)^2 + a(-3) + b = -86$

$2(-27) - (9) + a(-3) + b = -86$

$-54 - 9 - 3a + b = -86$

$-3a + b = -23$ **(2)**

Multiply equation **(2)** by -1 then add it to equation **(1)**

$2a + b = 2$

$\underline{3a - b = 23}$

$5a \quad = 25$

$a = 5$

When $a = 5$, $b = -8$

10 $\mathrm{f}(x) = 3x^3 + 2x^2 - px + q$

$\mathrm{f}(1) = 0$

So

$3(1)^3 + 2(1)^2 - p(1) + q = 0$

$3 + 2 - p + q = 0$

$p - q = 5$ **(1)**

$\mathrm{f}(-1) = 10$

So

$3(-1)^3 + 2(-1)^2 - p(-1) + q = 10$

$-3 + 2 + p + q = 10$

$p + q = 11$ **(2)**

Adding equations **(1)** and **(2)** gives

$p - q = 5$

$\underline{p + q = 11}$

$2p \quad = 16$

$p = 8$

When $p = 8$, $q = 3$